

Single Phase Induction Motor

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1 Single phase induction motor

Overview

The module first explains the kind of field produced by a single phase winding excited by a.c current and shows that it is equivalent to two rotating fields rotating in opposite directions. Development of equivalent circuit, torque- slip characteristic and performance evaluation. Production of starting torque by using auxiliary winding with phase splitting. These module has x units covering the following aspects.

Module Objectives

After going through this module, students will be able to

1. explain why no starting torque exist in a single phase induction motor.
2. prove that a standing pulsating magnetic field is equivalent to two rotating fields rotating in the opposite directions.
3. predict the frequencies of rotor currents when the motor runs at slip s .
4. comment on the relative strengths of forward and backward fields at different values of slip.
5. calculate forward, backward and net torque at slip s .
6. estimate the value of capacitance to maximize starting torque.
7. predict the performance of the motor.

1.1 Constructional features

Overview

Single phase induction motor has no starting torque however it can produce running torque while operating with a single winding. In contrast with a 3-phase induction motor, a single phase induction motor has two windings namely main and auxiliary (or starting) windings on the stator whose magnetic axis are in quadrature. As the name suggests, the auxiliary or the starting winding is kept energised only briefly, till the rotor attains sufficient speed. During running condition motor runs on main winding alone.

Objectives

After going through this unit, students will be able to:

1. explain why a single phase induction motor does not have starting torque but has a running torque.
2. explain the importance of auxiliary winding.

3. to conclude that auxiliary winding should be short time rated.
4. understand the function of the centrifugal switch connected in series with the auxiliary winding.

1.1.1 Suggested material to meet the objectives

Single phase induction motor

Single phase induction motors are manufactured in small sizes not exceeding 1HP or so. These belong to FHP (fractional horse power) category.

The rotor of such machines are of cage type only. Usually a capacitor is connected in series with the starting winding and paralleled with the main winding. This parallel combination is supplied from a single phase source. The presence of capacitor in the auxiliary winding creates a substantial amount of phase difference between the main and the auxiliary winding currents to give reasonable starting torque. Both the auxiliary winding and the capacitor are short time rated for obvious reasons in case of *capacitor start motor*. Also a *centrifuge switch* is connected in series with the auxiliary winding to facilitate its disconnection when machine attains about 75% of its rated speed. However, there are single phase induction motor where the *auxiliary* winding is permanently connected in the circuit along with capacitor. Figure 2 lists various types of single phase induction motors and figure 1 shows the representation and connection diagram of a capacitor start single phase induction motor.

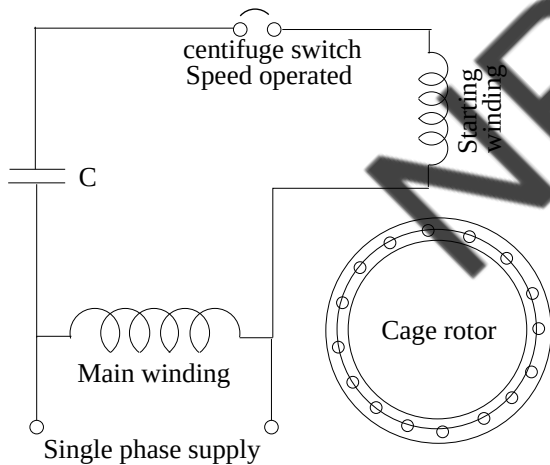


Figure 1: Single phase machine connection.

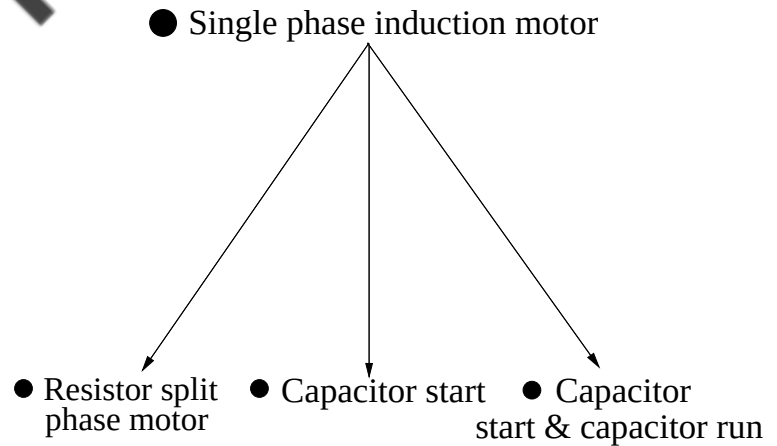


Figure 2: Types of single phase motor.

Starting torque of single phase induction motor

We know that a single phase winding excited from a single phase source produces a standing pulsating wave and not a rotating magnetic field as in a 3-phase induction motor. This pulsating field, at starting, can be shown to be equivalent to two identical rotating fields moving in the opposite directions at synchronous speed. The field which moves in the clockwise direction exerts torque in the clockwise direction and the field which moves in the counter clockwise direction exerts torque

in the clockwise direction. These two equal torques acting in opposite direction does not allow the rotor to have any net starting torque. Thus a single phase induction motor has no inherent starting torque.

Another nice way of explaining that a single phase induction motor cannot have a starting torque is to refer to figure 3.

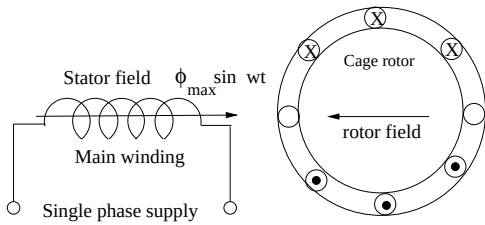


Figure 3: Stator & rotor field at starting.

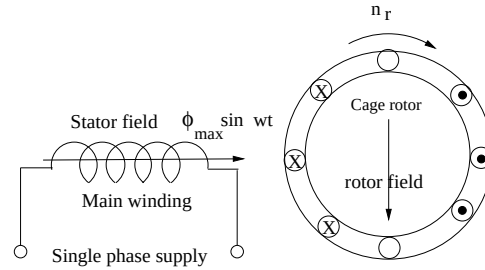


Figure 4: Rotor field due to rotational voltage.

Let at any time the stator field in the direction shown. Rotor being stationary, there will be statically induced (transformer) voltage in the rotor bars. As per Lenz's law the upper bars will carry cross current and the lower bars will carry dot currents. Therefore the direction of rotor field will be from right to left as shown. The axis of stator and rotor poles coincide and these two sets of poles are unable to produce any torque.

It is interesting to note that the single phase induction motor, however, can produce running torque while working with a single winding. This too can be explained nicely by referring to figure 4. Suppose the rotor is rotating in the clockwise direction with speed n_r . Then there will be rotational induced voltage ($= blv$) in the rotor bars and hence current. This time we see that rotor field due to rotational voltage will be acting in the vertical direction. Thus interaction of stator and rotor fields will give running torque.

1.1.2 Problems

1. Physically explain why does a single phase induction motor have no starting torque.
2. Physically explain why does a single phase induction motor have running torque.
3. What is the function of auxiliary winding? Why does auxiliary winding is short time rated?
4. What is the function of centrifugal switch and why is it necessary?

1.2 Double Revolving Field Theory

Overview

Understanding and development of equivalent circuit of a single phase induction motor can be carried out on the basis of double revolving field theory. It is shown that when the motor runs on main winding alone, the pulsating field produced by it can be resolved into two rotating fields, moving in opposite directions. So for each of these fields we can draw the equivalent circuit in the same way as was done for 3-phase induction motor.

Objectives

After going through this unit, students will be able to:

1. visualise that a pulsating field is equivalent to two rotating fields moving in opposite directions.
2. resolve the unbalanced currents in auxiliary winding ($\bar{I}_A = 0$) & in main winding $\bar{I}_M$ into a balanced +ve sequence currents and a balanced -ve sequence currents in terms of the main winding current.
3. appreciate that the forward field is produced by the +ve sequence currents and backward field is produced by the -ve sequence currents.
4. ascertain that if s is the slip corresponding to forward motor; the slip corresponding to backward field will be $(2 - s)$.

1.2.1 Suggested material to meet the objectives

Introduction

In this chapter we shall study the basic theory and the constructional features of single phase induction motor. Ratings of single phase induction motors are restricted to 1 HP, hence they are also called FHP motors. Unlike three phase induction motor, a single phase induction motor has no inherent starting torque however, it can have a running torque. The development of equivalent circuit of such a machine is somewhat involved compared to that of its three phase counterpart. The analysis of 1-phase induction motor can be carried out by *Double Field Revolving Theorem* or by *Cross Field Theorem*. Here we shall adopt double revolving field theory to Analyse the performance of single phase induction motor.

Principle of operation

The stator houses a distributed single phase winding. Such a winding when excited by a single phase source, a standing pulsating field along the air gap results. We show below that this pulsating field is equivalent to two rotating fields moving in the opposite directions. If the exciting current is $i = I_{max} \sin \omega t$, then the mmf distribution along the air gap is given by,

$$f_\theta = F_{max} \cos \omega t \cos \theta$$

where θ is the electrical space angle and F_{max} is proportional to the effective number of turns and I_{max} . The above equation can easily be simplified to

$$f_\theta = \frac{1}{2} F_{max} \cos (\omega t + \theta) + \frac{1}{2} F_{max} \cos (\omega t - \theta)$$

The first term corresponds to the *forward* rotating field while the second term corresponds to the *backward* rotating field. The speed of both the fields are same and equal to ω . In other words we can say that a single phase distributed winding if excited from a single phase supply produces a standing pulsating field which is equivalent to two equally strength fields rotating in the opposite directions with equal velocity. This is the basis of the double revolving field theory. The rotor of 1

phase induction motor is cage type. In case of 3-phase motor, single rotating field is produced and the cage rotor experiences torque along the direction of the rotating field. A cage rotor requires to interact with a rotating field for torque production. So far torque production is concerned, cage does not question how a rotating field is produced. It may be produced by a balanced 3-phase winding or a balanced 2-phase winding and so on. At the time of starting, both the fields produce torques of equal strength but in the opposite directions; hence no starting torque is possible in a single phase induction motor. The equivalence is diagrammatically shown in figure 5

Let us look at the starting phenomenon from a different angle. The cage rotor behaves as a short circuited secondary of a transformer when single phase stator winding is energised. Thus secondary or the rotor winding carries current and produces its own field the direction of which is aligned along the direction of the stator field as shown in the figure 6.

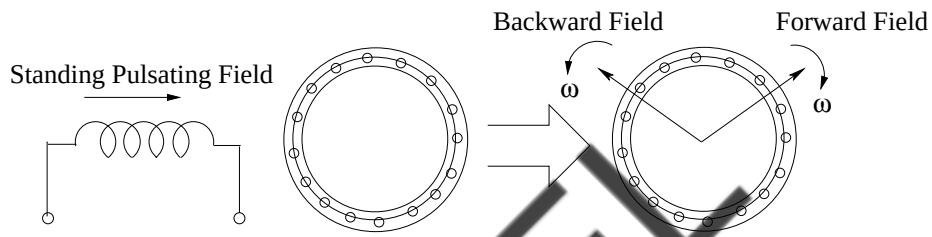


Figure 5: Pulsating field: equivalent to rotating fields..

Obviously the interaction of these two fields can not produce any torque. Hence starting torque must be zero.

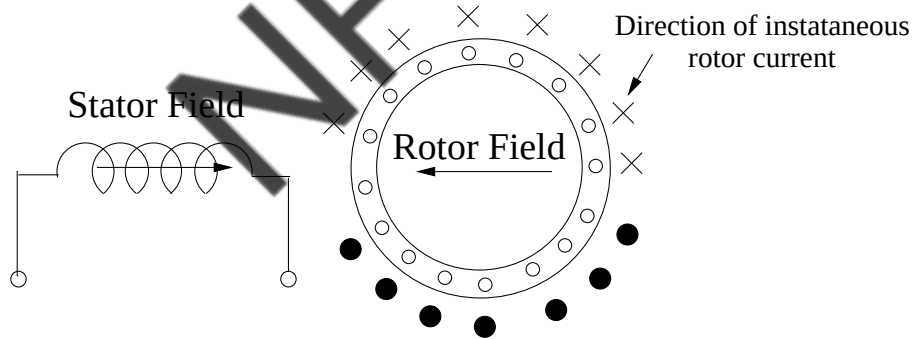


Figure 6: Stator and rotor fields.

In order therefore to have starting torque additional arrangements must be provided in the machine. One of them and which is very popularly used is to have another additional winding on the stator displaced in space from the main winding by 90° electrical angle. The derivation in the following section will bring out the essence of this method.

1.2.2 Problems

1. Show that the nature of the field produced by a single phase winding excited by a single phase source produces a standing pulsating field along the air.

2. Prove that, a pulsating field is equivalent to two rotating field moving in opposite directions.
3. Comment on the speed and magnitudes of the forward and backward field at the time of starting a single phase induction motor.
4. A standing pulsating field is given by $f = 1.8 \sin 157t \sin \theta$ units. Calculate the magnitudes and speeds of the forward and backward fields.

1.3 Equivalent Circuit and torque-slip Characteristics

Overview

Development of equivalent circuit of a single phase induction motor running at any slip s and on main winding alone is discussed in this module. To begin with two separate equivalent circuits are developed one corresponding to forward field and the other corresponding to backward field. Finally these two circuits are interconnected suitably to get a single equivalent circuit where applied voltage is the supply voltage and current drawn from the supply is the main winding current. Once equivalent circuit is obtained, calculation of torque, mechanical power developed, power factor at which the motor is operating and efficiency can be calculated.

Objectives

After going through this unit, students will be able to:

1. separately draw the equivalent circuit corresponding to forward field and that due to backward field.
2. make out that the two circuits are to be series connected and parameters values divided by factor of two.
3. conclude that the strengths of the forward and backward fields are same at the time of starting ($s = 1$) but at any other slip s , strength of forward field will be much higher.
4. use equivalent circuit to find out forward torque, backward torque, net torque, output power.
5. sketch torque-slip characteristic.

1.3.1 Suggested material to meet the objectives

Development of equivalent circuit

So far we have restricted our attention to the starting of single phase induction motor and seen that how starting torque can be provided with the help of an auxiliary winding. However, the motor will finally run on main winding alone and supply the mechanical load with auxiliary winding completely disconnected. In this section we shall discuss how to analyse the performance of the machine when it runs with a slip of s . Similar to 3-phase induction motor, here also we shall try to develop the equivalent circuit of single phase induction motor and predict its performance during running condition.

The philosophy of obtaining the equivalent circuit will center around the fact that during running condition the machine can be considered to be operating as a unbalanced two phase motor. Next logical step will be to replace this unbalanced situation by one balanced +ve sequence and another balanced -ve sequence system.

A single phase induction motor running on main winding can be considered to be an unbalanced two phase system with main winding current to be I_M and auxiliary current to be $I_A = 0$. Let,

$$\begin{aligned}\bar{I}_M &= \bar{I}_{M1} + \bar{I}_{M2} \\ \bar{I}_A &= \bar{I}_{A1} + \bar{I}_{A2} = 0\end{aligned}\tag{1}$$

Where, $\bar{I}_{M1}$ and $\bar{I}_{A1}$ forms a balanced +ve sequence system and $\bar{I}_{M2}$ and $\bar{I}_{A2}$ forms a balanced -ve sequence system. Then the following relations hold good:

$$\begin{aligned}\bar{I}_{A1} &= j\bar{I}_{M1} \\ \bar{I}_{A2} &= -j\bar{I}_{M2}\end{aligned}\tag{2}$$

Adding the above equations we get $\bar{I}_{M1} = \bar{I}_{M2}$ and note that $\bar{I}_{A1} = -\bar{I}_{A2}$. From the above equations we further get,

$$\begin{aligned}\bar{I}_{M1} &= \bar{I}_{M2} = \frac{\bar{I}_M}{2} \\ \bar{V}_M &= \bar{V}_{M1} + \bar{V}_{M2}\end{aligned}\tag{3}$$

Since $\bar{I}_{M1}$ & $\bar{I}_{A1}$ and $\bar{I}_{M2}$ & $\bar{I}_{A2}$ are independently balanced two phase systems, we can draw the *per phase equivalent circuit* of each of them in the same line as that of a balanced three phase induction motor. The direction of rotation of the fields obviously will be in the opposite sense. Let us call field produced by $\bar{I}_{M1}$ & $\bar{I}_{A1}$ to be *forward field* and the other field produced by $\bar{I}_{M2}$ & $\bar{I}_{A2}$ be called *backward field*. Also suppose the rotor moves with a speed of n_r along the same direction of the forward field. Then the slip of the motor corresponding to forward field will be $s = \frac{n_s - n_r}{n_s}$ and slip due to backward field will be $s_b = \frac{-n_s - n_r}{-n_s} = 2 - s$. With this in mind the equivalent circuit per phase of the +ve sequence motor and the -ve sequence motor can be easily drawn and shown in figure 7.

The above two circuits in figure 10 can be connected in series as $\bar{I}_{M1} = \bar{I}_{M2}$. The applied voltage across the series combination will be obviously the applied voltage to the main winding, $\bar{V}_M = (\bar{V}_{M1} + \bar{V}_{M2})$. This is shown in the figure 8. The important thing to be noted here is that the air gap powers calculated from this circuit gives per phase values and have to be multiplied by a factor of 2 to get the total air gap power or the torque at any slip.

For single phase motor running on main winding alone, we can record the applied voltage $\bar{V}_M$ and the main winding current $\bar{I}_M$. In fact it would have been nice if the equivalent circuit could be modified such that $\frac{\bar{I}_M}{2}$ is changed to $\bar{I}_M$. In fact this can be done by representing the each

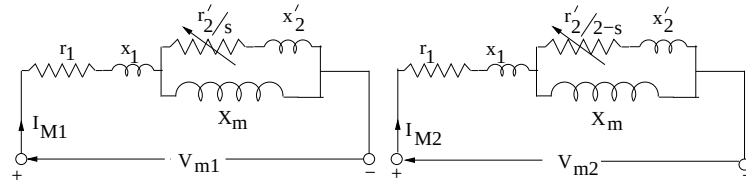


Figure 7: Per phase equivalent circuit of +ve and -ve sequence motors.

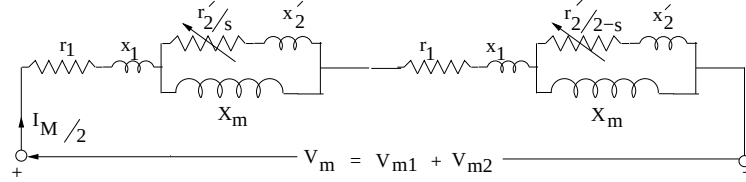


Figure 8: +ve & -ve sequence circuits in series.

parameter of the equivalent circuit of figure 10 by 2. The final equivalent circuit is shown in figure 9.

The power level of the final equivalent circuit becomes obviously two times. Hence power calculated in the resistors $\frac{r'_2}{2s}$ and $\frac{r'_2}{2(2-s)}$ will straight away give the respective air gap powers for the forward and the backward field. The expression for the torque when the machine runs with a slip s , is given by :

$$\begin{aligned}
 T &= \frac{\text{Net air gap power}}{2\pi n_s} \\
 &= \frac{\text{Air gap power due to forward field}}{2\pi n_s} - \frac{\text{Air gap power due to backward field}}{2\pi n_s} \\
 &= \frac{1}{2\pi n_s} \left(\frac{I_f^2 r'_2}{2s} - \frac{I_b^2 r'_2}{2(2-s)} \right) \quad (4)
 \end{aligned}$$

The torque slip curve for the forward field, backward field and the resultant field are shown in the figure 10. It can easily be seen that at $s = 1$ resultant torque is zero while during forward running condition $T_f > T_b$, hence running torque exists. The rotor may accelerate in either forward or backward direction depending upon in which direction the starting torque has been provided by an external agency or by the auxiliary winding.

It should be noted that when the runs with a small slip, $\frac{r'_2}{2s} \gg \frac{r'_2}{2(2-s)}$ making $E_f \gg E_b$. Hence the strength of the forward field is much higher than the backward field. Thus during running condition two fields are rotating in the opposite direction with same velocity but the forward field is much stronger than the backward field resulting into a net forward torque. In effect the locus of the tip of the resultant field vector is an *ellipse*. Let us start counting time from the instant when M_f and M_b overlap each other in the horizontal position i.e., along the direction of the x-axis of figure 11.

At any arbitrary time t , the coordinates of the forward and backward fields are respectively $(M_f \cos \omega t, -M_f \sin \omega t)$ and $(M_b \cos \omega t, M_b \sin \omega t)$. Therefore x and y coordinates of the resultant field are given by:

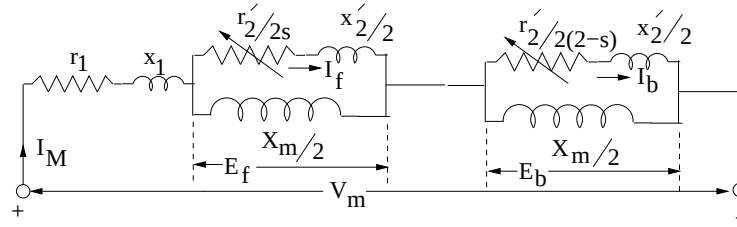


Figure 9: Final equivalent circuit.

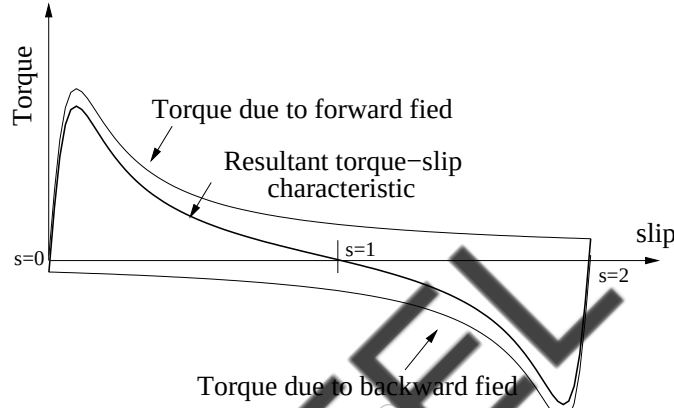


Figure 10: T vs. slip curves for forward, backward and resultant fields.

$$\begin{aligned} x &= M_f \cos \omega t + M_b \cos \omega t \\ y &= M_b \sin \omega t - M_f \sin \omega t \end{aligned} \quad (5)$$

Eliminating the $\sin \omega t$ and $\cos \omega t$ from the above equations we get the locus of the resultant field as:

$$\left(\frac{x}{M_f + M_b} \right)^2 + \left(\frac{y}{M_f - M_b} \right)^2$$

The locus is an ellipse with major and minor axes being $2M_f + M_b$ and $2M_f - M_b$. It may be recalled that in case of a 3 phase induction motor, the strength of the air gap field is time invariant and its locus is a circle.

Frequency of rotor current during running condition

A single phase induction motor running with slip s will have rotor currents of two frequencies namely sf and $(2-s)f$, where f is the stator supply frequency. Suppose the stator supply frequency $f = 50\text{Hz}$ and motor is running with a slip of 4 %. Rotor current frequency due to forward field is $0.04 \times 50 = 2\text{Hz}$ and due to backward field $(2 - 0.04) \times 50 = 98\text{Hz}$. Typical rotor current waveform is shown in figure 12 assuming 98 Hz component of current to be 10 % that of 2 Hz component of the current.

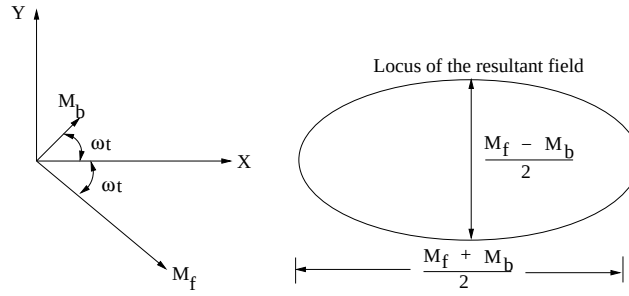


Figure 11: Locus of the resultant field.

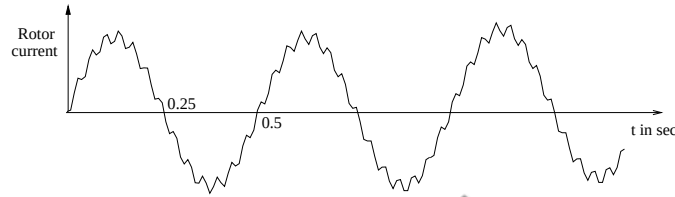


Figure 12: Typical rotor current waveform.

2 Expression for the starting torque

Let the inherent impedances offered to the supply by the main winding and the auxiliary windings be $\bar{Z}_M = r_M + jx_M$ and $\bar{Z}_A = r_A + jx_A$ at stationary condition i.e., at $s = 1$

The expression for starting torque is derived rather elegantly by replacing the main and auxiliary winding currents into its +ve and -ve sequence components. The idea is to consider I_M and I_A as unbalanced two phase currents and break them up into +ve and -ve sequence components as follows:

$$\begin{aligned}\bar{I}_M &= \bar{I}_{M1} + \bar{I}_{M2} \\ \bar{I}_A &= \bar{I}_{A1} + \bar{I}_{A2}\end{aligned}\tag{6}$$

The positive and negative sequence auxiliary winding currents can be expressed in terms of positive and negative sequence components of the main winding currents as shown below.

$$\begin{aligned}\bar{I}_{A1} &= j\bar{I}_{M1} \\ \bar{I}_{A2} &= -j\bar{I}_{M2}\end{aligned}\tag{7}$$

Therefore auxiliary current becomes, $\bar{I}_A = j\bar{I}_{M1} - j\bar{I}_{M2}$. After doing algebraic manipulations, +ve and -ve sequence of main winding currents can be expressed as,

$$\begin{aligned}2\bar{I}_{M1} &= \bar{I}_M - j\bar{I}_A \\ 2\bar{I}_{M2} &= \bar{I}_M + \bar{I}_A\end{aligned}\tag{8}$$

Now $\bar{I}_{M1}$ and $\bar{I}_{A1}$ form a balanced 2-phase system giving +ve torque, while $\bar{I}_{M2}$ and $\bar{I}_{A2}$ also form a balanced 2-phase system but giving -ve torque. Thus at starting with $s = 1$,

$$\begin{aligned} \text{+ve sequence torque} &= T_+ = 2I_{M1}^2 r'_2 \\ \text{-ve sequence torque} &= T_- = 2I_{M2}^2 r'_2 \\ \text{Net Starting torque} &= T_{st} = 2(I_{M1}^2 - I_{M2}^2)r'_2 \end{aligned} \quad (9)$$

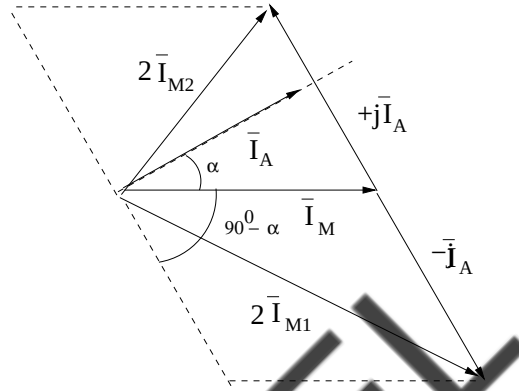


Figure 13: Phasor diagram relating actual & sequence component currents.

Now from the phasor diagram shown in figure 13, we can write:

$$\begin{aligned} (2I_{M1})^2 &= I_M^2 + I_A^2 + 2I_M I_A \cos(90^\circ - \alpha) \\ (2I_{M2})^2 &= I_M^2 + I_A^2 + 2I_M I_A \cos(90^\circ + \alpha) \end{aligned} \quad (10)$$

Subtracting the above two we get,

$$I_{M1}^2 - I_{M2}^2 = I_M I_A \sin \alpha$$

So putting this value in the expression of the starting torque, we get,

$$T_{st} = r'_2 2I_M I_A \sin \alpha \text{ (in synchronous watt)}$$

In the above derivation it has been assumed that the number of main and auxiliary turns are equal so that each of the +ve and -ve sequence motor could be considered as balanced two phase motors. However, if the turns ratio $\frac{N_A}{N_M} = a$, is other than unity, then the starting torque expression can be easily found out by replacing I_A by aI_A . Hence final expression for starting torque becomes,

$$T_{st} = \frac{r'_2}{2\pi n_s} 2a I_M I_A \sin \alpha$$

2.1 Unit-3 Problems

1. Comment on the magnitudes and speeds of the forward & backward fields during running condition of a single phase induction motor.

2. A 50 Hz, single phase induction motor runs with slip of 4%. Comment and calculate the frequency of rotor current.
3. Sketch the locus of the tip of the air gap field of a single phase induction motor for the following cases:
 - (a) at the time of starting.
 - (b) when the machine is running steadily at a constant slip.
4. Unbalanced two phase currents are given as $i_1 = 5 \sin \omega t$ A and $i_2 = 2 \sin(\omega t + 70^\circ)$ A. Obtain the +ve and -ve sequence components and write their expressions.
5. Show that to maximise starting torque, one has to maximise $I_A \sin \theta$ where I_A is the auxiliary winding current and θ is the phase angle difference between $\bar{I}_A$ and $\bar{I}_M$. Main winding current being $\bar{I}_M$.

2.2 Phase splitting in Single Phase Induction Motor

Overview

A single phase induction motor running on single (main) winding alone can not produce any starting torque. Another winding called starting or auxiliary winding whose magnetic axis in space is at right angles to that of main winding, is provided on the stator so that the motor can have starting torque. The starting torque is proportional to the sine of the angle between main and auxiliary winding currents and also to the magnitudes of the main & auxiliary currents.

For capacitor start motor, the auxiliary winding gets disconnected after the motor attains a speed of about 75% of the synchronous speed. So auxiliary winding is short time rated having thinner conductor section. In case of capacitor run motor auxiliary winding continues to be in the circuit continuously and is never disconnected. However, the values of capacitor connected in series with the auxiliary winding are different during starting and running conditions. In this module, selection of starting capacitor at the time of starting is discussed.

Objectives

After going through this unit, students will be able to:

1. understand the term phase splitting and its importance
2. derive an expression for starting torque and identify the factors on which it depends.
3. draw circle diagram for auxiliary winding current when (i) capacitance is varied and (ii) resistance in auxiliary circuit is varied.
4. select capacitance value for (i) maximum starting torque; (ii) maximising the ratio of maximum starting torque : supply current.

2.3 Suggested material to meet the objectives

How to maximise starting torque?

Resistor split motor

After deciding upon the type phase splitting chosen, one can estimate the value of the external components (resistor or capacitor) to be connected in series with the auxiliary winding in order to have (i) maximum starting torque or (ii) main and auxiliary winding current in quadrature or (iii) the ratio between the starting torque & the total starting current maximised. Obviously maximising starting torque essentially means maximising the product $I_A \sin \theta$. Consider first the resistor split motor where we would like to find out what external resistance should be connected in series with the auxiliary winding so as to get maximum starting torque. Refer to the figures 14 where the locus of the tip of the auxiliary current phasor is shown for any arbitrary resistance. We know the tip of the current phasor will lie on a circle of diameter $\frac{V}{x_A}$ as the value of auxiliary winding resistance is changed. The I_M phasor

lags the supply voltage by its fixed power factor angle θ_M while I_A phasor lags by θ_A . The perpendicular length PQ is the measure of $I_A \sin \theta$. Hence the value of the starting torque in this case will be:

$$T = \frac{r_2'}{2\pi n_s} 2a I_A I_M \sin \theta = \frac{r_2'}{2\pi n_s} 2a I_M (PQ)$$

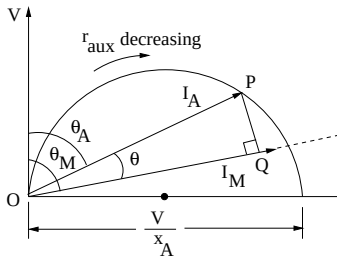


Figure 14: Position of I_A for arbitrary r_A

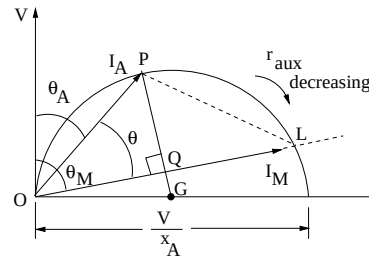


Figure 15: Position of I_A for maximum torque.

The length PQ will be maximum if the tip of I_A (i.e., the point P) lies in such a way that the perpendicular drawn on the I_M phasor will pass through the center G of the the circle as shown in the figure 15. Obviously,

$$\begin{aligned} \angle POQ &= \angle PLQ \\ \theta_M - \theta_A &= \theta_A \\ \theta_A &= \frac{\theta_M}{2} \\ \tan \theta_A &= \tan \frac{\theta_M}{2} \\ \tan \theta_A &= \frac{\sin \theta_M}{1 + \cos \theta_M} \end{aligned}$$

$$\begin{aligned}\frac{x_A}{r_A} &= \frac{x_M}{1 + r_M} \\ r_A + r_{ext} &= \frac{(Z_M + r_M)x_A}{x_M}\end{aligned}\quad (11)$$

Hence r_{ext} can be calculated. external resistance to be connected can be easily found out by subtracting the *auxiliary winding* resistance from the calculated r_A . One can also derive an expression for the starting torque in terms of supply voltage and other machine parameters as follows:

$$\begin{aligned}T &= \frac{r_2'}{2\pi n_s} 2a I_M (PQ) \\ &= \frac{r_2'}{2\pi n_s} 2a \frac{V}{z_M} (PG - GQ) \\ &= \frac{r_2'}{2\pi n_s} 2a \frac{V}{z_M} \left(\frac{V}{2x_A} - \frac{V}{2x_A} \cos \theta_M \right) \\ &= \frac{r_2'}{2\pi n_s} 2a \frac{V}{z_M} \frac{V}{2x_A} \left(1 - \frac{r_M}{z_M} \right) \\ &= \frac{r_2' a}{2\pi n_s} \left(\frac{V}{z_M} \right)^2 \frac{(z_M - r_M)}{x_A}\end{aligned}\quad (12)$$

2.3.1 Capacitor split motor

In this case external capacitor is connected in series with the auxiliary winding so as to make the auxiliary current lead the supply voltage. In this case also the tip of I_A will lie on a circle of diameter $\frac{V}{r_A}$. The figure 16 shows the position of I_A phasor for an arbitrary value of C . The starting torque will be proportional to the length $PQ (= I_A \sin \theta)$.

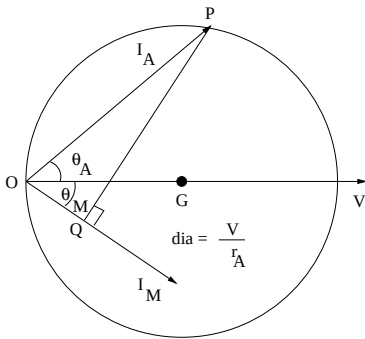


Figure 16: Position of I_A for arbitrary C

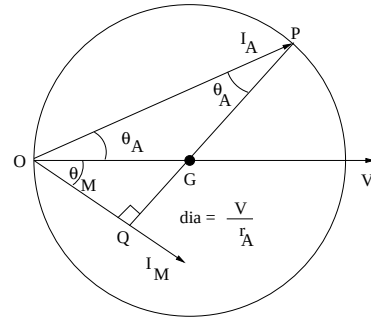


Figure 17: Position of I_A for maximum torque.

To have maximum starting torque, the line PQ must pass through G the center of the circle as shown in figure 17. From simple geometry: $\angle POG = \angle GPO = \theta_A$ and $2\theta_A + \theta_M = 90^\circ$. Therefore,

$$\tan \theta_A = \sqrt{\frac{1 - \cos 2\theta_A}{1 + \cos 2\theta_A}}$$

$$\begin{aligned} &= \sqrt{\frac{1 - \sin \theta_M}{1 + \sin \theta_M}} \\ &= \sqrt{\frac{z_M - x_M}{z_M + x_M}} = \frac{r_M}{z_M + x_M} \end{aligned}$$

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$$\begin{aligned}
\text{but , } \tan \theta_A &= \frac{x_C - x_A}{r_A} \\
\therefore \frac{x_C - x_A}{r_A} &= \frac{r_M}{z_M + x_M} \\
x_C &= x_A + \frac{r_M}{z_M + x_M}
\end{aligned} \tag{13}$$

In the above equation it may be noted that x_A is the inherent inductive reactance of the auxiliary winding. An equation for the torque can also be found out in terms of the supply voltage and various parameters as shown below:

$$\begin{aligned}
T &= \frac{r_2'}{2\pi n_s} 2a I_M (PQ) \\
&= \frac{r_2'}{2\pi n_s} 2a \frac{V}{z_M} (PG + GQ) \\
&= \frac{r_2'}{2\pi n_s} 2a \frac{V}{z_M} \left(\frac{V}{2r_A} + \frac{V}{2r_A} \sin \theta_M \right) \\
&= \frac{r_2'}{2\pi n_s} 2a \frac{V}{z_M} \frac{V}{2r_A} \left(1 + \frac{x_M}{z_M} \right) \\
&= \frac{r_2' a}{2\pi n_s} \left(\frac{V}{z_M} \right)^2 \frac{(z_M + x_M)}{r_A}
\end{aligned} \tag{14}$$

Thus we see that the the optimum values of resistor or capacitor may be found out depending upon the kind phase splitting adopted in order to maximise the starting torque. In fact instead of using formulae given above, one can find out the optimum values of the parameters and as well as the torque by purely graphical methods. There may be considerations other than maximising the starting torque. For example one may be interested to select a starting capacitance for which I_A and I_M will be in quadrature or one may be interested to maximise the ratio $\frac{T}{I_{starting}}$. The later consideration is important because it states that how every amepere drawn from the supply is utilised for developing the starting torque. Looking at figure 18, we note maximisation of $\frac{T}{I_{starting}}$ means maximisation of the ratio of the lengths PN and ON. It is assumed that we are adopting capacitor split phasing. From geometrical consideration it can be shown that the ratio will be maximum when the length OP is tangent to the circle at point P. In other words the tip of the total starting current phasor I_{st} will be the touching point. Under this condition positions of different phasors are shown in the figure 19.

Similar diagrams can also be drawn for resistor split phase motor.

2.3.2 Problems

1. Find out an expression for capacitive reactance needed in the auxiliary winding to make auxiliary and main winding current in quadrature at the time of starting.
2. Sketch the current locus of single phase induction motor at starting and answer the following:

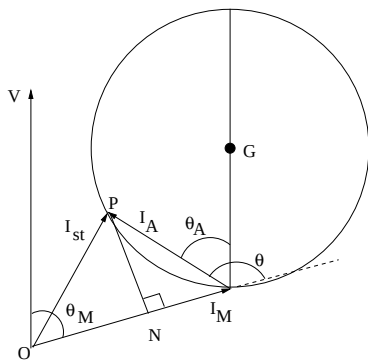


Figure 18: Arbitrary Position of I_A

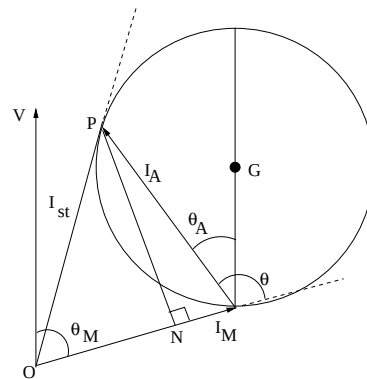


Figure 19: Position of I_A , for $\frac{T}{I_{st}}$ to be maximum

- How do you find out extra resistance needed in the auxiliary winding to maximise starting torque for resistor phase splitting?
- How do you find out capacitance needed in the auxiliary winding to maximise starting torque for capacitor phase splitting?
- How do you find out capacitance needed in the auxiliary winding to maximise the ratio of *starting torque:current drawn from the supply* for capacitor phase splitting?